Solution for Interest Rate and Payment Amount — *Bret D. Whissel*

It is possible to determine the interest rate *and* periodic payment amount of an established amortization schedule if one knows only the original amount of the loan, the principal remaining, the original length of the loan term, and the number of payments which have already been made. So using the following variable definitions,

P	The principal borrowed
N	The total number of scheduled payments
i	The fractional (periodic) interest rate
R	The principal remaining after r payments
r	Some number of payments such that $0 < r < N$
x	The regular periodic payment

we're trying to find a solution for both x and i when the other variables are known.

From my "Derivation of Amortization" document, assuming we will not be dealing with a balloon payment, we have the following equation for determining the periodic payment amount x (cf. Eq. 6):

$$x = \frac{iP(1+i)^N}{(1+i)^N - 1}. \qquad [6*]$$

From my "Additional Payment Tweaks" document, I found this equation for determining the principal remaining (R) after r payments have been made according to schedule (Eq. 12):

$$R = P - (x - iP)\frac{(1+i)^r - 1}{i}. \qquad [12]$$

If we rewrite Eq. 12 in terms of x, we get

$$x = \frac{i(P-R)}{(1+i)^r - 1} + iP, \qquad (14)$$

and then we can set Eqs. 6* and 14 equal:

$$\frac{iP(1+i)^N}{(1+i)^N - 1} = x = \frac{i(P-R)}{(1+i)^r - 1} + iP. \qquad (15)$$

Begin simplifying by dividing through by the common multiplier i:

$$\frac{P(1+i)^N}{(1+i)^N - 1} = \frac{P-R}{(1+i)^r - 1} + P$$

$$P\left[\frac{(1+i)^N}{(1+i)^N - 1} - 1\right] = \frac{P-R}{(1+i)^r - 1}$$

Now attempt to isolate the i terms to get

$$[(1+i)^r - 1]\left[\frac{(1+i)^N}{(1+i)^N - 1} - 1\right] = \frac{P-R}{P}.$$

Multiplying through and simplifying again results in this arrangement of terms,

$$1 - \frac{P-R}{P} = \frac{(1+i)^N - (1+i)^r}{(1+i)^N - 1}. \qquad (16)$$

Given the generally high order of N, finding i will require numerical methods, though it should be fairly easy to find i to arbitrary precision with a computer program or spreadsheet using this equation.

Plugging the resulting value for i into Eq. 6* or Eq. 14 will give an answer for x, the periodic payment.

References include "A Derivation of Amortization" at `http://www.bretwhissel.net/amortization/amortize.pdf` and "Some Additional Payment Tweaks" at `http://www.bretwhissel.net/amortization/finextra.pdf`.